

# M.Tech Data Science

## Arithmetics Mean

1. Let  $A_1, A_2$  be two AM's and  $G_1, G_2$  be two GM's between  $a$  and  $b$ , then  $(A_1 + A_2) / G_1 G_2$  is equal to

- a)  $(a+b) / 2ab$
- b)  $2ab/(a+b)$
- c)  $(a+b)/(ab)$
- d) None of the mentioned

Answer: c

Explanation:  $A_1 + A_2 = a + b$ ,  $G_1 G_2 = ab$ .

2. The series  $a, (a+b)/2, b$  is in \_\_\_\_\_

- a) AP
- b) GP
- c) HP
- d) None of the mentioned

Answer: a

Explanation:  $(a+b)/2$  is AM between  $a, b$ . Hence the series is in AP.

3. The series  $a, (ab)^{1/2}, b$  is in \_\_\_\_\_

- a) AP
- b) GP
- c) HP
- d) None of the mentioned

Answer: b

Explanation:  $(ab)^{1/2}$  is GM between  $a, b$ . Hence series is in GP.

4. If  $A$  and  $G$  be the A.M and G.M between two positive number then the numbers are  $A + (A_2 - G_2)^{1/2}, A - (A_2 - G_2)^{1/2}$ .

- a) True
- b) False

Answer: a

Explanation: The equation having its roots as given equation is  $x^2 - 2Ax + G_2 = 0$  which implies  $x = A \pm (A_2 - G_2)^{1/2}$ .

5. If one geometric mean  $G$  and two arithmetic mean  $A_1, A_2$  are inserted between two numbers, then  $(2A_1 - A_2) (2A_2 - A_1)$  is equal to \_\_\_\_\_

- a)  $2G$
- b)  $G$
- c)  $G^2$
- d) None of the mentioned



Answer: c

Explanation: Let a and b be two numbers then,  $G = (ab)^{1/2}$ ,  $A_1 = (2a+b)/3$ ,  $A_2 = (a+2b)/3$ ,  $(2A_1 - A_2) = a$ ,  $(2A_2 - A_1) = b$ ,  $(2A_1 - A_2)(2A_2 - A_1) = G^2$ .

6. State whether the given statement is true or false.

**AM  $\leq$  GM.**

- a) True
- b) False

Answer: b

Explanation: Arithmetic Mean is always greater or equal to the geometric mean.

7. If between two numbers which are root of given equation.  $x^2 - 18x + 16 = 0$ , a GM is inserted then the value of that GM is?

- a) 4
- b) 5
- c) 6
- d) 16

Answer: a

Explanation:  $x^2 - 2Ax + G^2 = 0$ , here  $G^2 = 16$  and therefore  $G = 4$ .

8. If  $a_1, a_2, a_3$  are in arithmetic as well as geometric progression then which of the following is/are correct?

- a)  $2a_2 = a_1 + a_3$
- b)  $a_2 = (a_1 a_3)^{1/2}$
- c)  $a_2 - a_1 = a_3 - a_2$
- d) All of the mentioned are correct

Answer: d

Explanation:  $a_2$  is AM, GM between  $a_1, a_3$ , also the series is in AP so common differences should be the same.

9. If  $a_1, a_2, a_3$  are in GP then  $1/a_1, 1/a_2, 1/a_3$  are in \_\_\_\_\_

- a) AP
- b) GP
- c) HP
- d) None of the mentioned

Answer: b

Explanation: Let the terms be  $ar, a, a/r$  then reciprocals are  $1/(ar), 1/a, r/a$ . Still the terms are in GP.

10. If  $a_1, a_2, a_3, \dots$  are in AP then if  $a_7 = 15$ , then the value of common difference that would make  $a_2, a_7, a_{12}$  greatest is?

- a) 2
- b) 0
- c) 4
- d) 9

Answer: b

Explanation: Let  $d$  be a common difference of the AP. Then,  $a_2 a_7 a_{12} = (15 - 5d)(15)(15 + 5d) = 375(9 - d^2)$  For maximum value  $d=0$ .

### **Part -II**

1. Let the sequence be  $1 \times 2, 3 \times 2^2, 5 \times 2^3, 7 \times 2^4, 9 \times 2^5, \dots$  then this sequence is \_\_\_\_\_

- a) An arithmetic sequence
- b) A geometric progression
- c) Arithmetico-geometric progression
- d) None of the mentioned

Answer: c

Explanation: If  $a_1, a_2, \dots$  are in AP and  $b_1, b_2, \dots$  are in GP then  $a_2 b_2, a_3 b_3, \dots$  are in AGP.

2. Let the sequence be  $1 \times 2, 3 \times 2^2, 5 \times 2^3, 7 \times 2^4, 9 \times 2^5, \dots$  then the next term of this AGP is given by \_\_\_\_\_

- a)  $10 \times 2^6$
- b)  $10 \times 2^7$
- c)  $11 \times 2^6$
- d) None of the mentioned

Answer: c

Explanation: Since here  $a_1, a_2, \dots$  are in AP and  $b_1, b_2, \dots$  are in GP then  $a_2 b_2, a_3 b_3, \dots$  are in AGP thus  $a_n = 11$  and  $b_n = 2^6$ .

3. The sum of the first  $n$  natural numbers is given by \_\_\_\_\_

- a)  $n(n+1)/2$
- b)  $n(n-1)/2$
- c)  $n^2(n+1)/2$
- d) None of the mentioned

Answer: a

Explanation:  $1 + 2 + 3 + 4 + \dots + n = (n/2)(1 + n)$  Since this is AP.

4. The sum of square of the first  $n$  natural numbers is given by \_\_\_\_\_

- a)  $n(n+1)(2n+1)/6$
- b)  $n(n-1)/2(2n+1)$
- c)  $n^2(n+1)(2n+1)/6$
- d) None of the mentioned

Answer: a

Explanation:  $1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = n(1+n)(2n+1)/6$ .

5. The sum of cubes of the first  $n$  natural numbers is given by \_\_\_\_\_

- a)  $\{n(n+1)/2\}^2$
- b)  $\{n(n-1)/2\}^2$

- c)  $\{n^2(n+1)/2\}_2$   
d) None of the mentioned

Answer: a

Explanation:  $1_3 + 2_3 + 3_3 + 4_3 + \dots + n_3 = \{n(n+1)/2\}_2$ .

6. The series 1, 1, 1, 1, 1..... is not an AGP.

- a) True  
b) False

Answer: b

Explanation: Since 1, 1, 1, 1, 1..... is in Ap and in Gp as well, Therefore the given sequence is also an AGP.

7. If in an AGP the common ratio of GP is 1 then that sequence becomes an AP sequence.

- a) True  
b) False

Answer: a

Explanation: In AGP sequence if  $r = 1$ , then terms are  $ab, (a+d)b, (a+2d)b, \dots$  and so on thus it is AP with common difference  $bd$ .

8. The sequence 1, 1, 1, 1, 1.... is?

- a) Absolutely summable  
b) Is not absolutely summable  
c) Can't say  
d) None of the mentioned

Answer: b

Explanation: For limit  $n$  tending to infinity the sum also tends to infinity and thus it is not summable.

9. Which of the following is a Triangular number series?

- a) 1, 3, 6, 9, 12, 15.....  
b) 1, 3, 6, 10, 15, 21.....  
c) 1, 6, 12, 18, 24.....  
d) none of the mentioned

Answer: b

Explanation: In triangular number sequence  $i$ th term is previous term +  $i$ , with first term as 1.

10. Which of the following is a fibonacci series?

- a) 0, 1, 2, 3, 4.....  
b) 0, 1, 1, 2, 3, 5.....  
c) 10, 12, 14, 16.....  
d) none of the mentioned

Answer: b

Explanation: Fibonacci series is formed by adding previous two term starting from 0 and 1.

### Harmonic Mean

1. If  $a_1, a_2, \dots$  are in AP then  $a_1^{-1}, a_2^{-1}, \dots$  are in \_\_\_\_\_

- a) An arithmetic sequence
- b) A geometric progression
- c) Arithmetico-geometric progression
- d) None of the mentioned

Answer: d

Explanation: If  $a_1, a_2, \dots$  are in AP, then  $a_1^{-1}, a_2^{-1}, \dots$  are in Harmonic Progression.

2. The ninth term of  $\sqrt[3]{3}, \sqrt[3]{7}, \sqrt[3]{11}, \sqrt[3]{15}, \sqrt[3]{19}, \dots$  is given by?

- a)  $\sqrt[3]{35}$
- b)  $\sqrt[3]{36}$
- c)  $\sqrt[3]{39}$
- d) None of the mentioned

Answer: a

Explanation: Since here  $a_1^{-1}, a_2^{-1}, \dots$  are in AP thus  $a_9 = 3 + (9-1)4 = 35$ ,  $\sqrt[3]{35}$  is 9th term of the series.

3. If for some numbers  $a$  and  $d$ , if the first term is  $\frac{1}{a}$ , second term is  $\frac{1}{(a+d)}$ , third term is  $\frac{1}{(a+2d)}$  and so on, then 5th term of the sequence is?

- a)  $a+4d$
- b)  $a-4d$
- c)  $\frac{1}{(a+4d)}$
- d) none of the mentioned

Answer: c

Explanation: The given sequence will form HP, thus 5th term will be  $(a+(5-1)d) - 1$ .

4. If  $a, b, c$  are in hp then  $a^{-1}, b^{-1}, c^{-1}$  are in \_\_\_\_\_

- a) GP
- b) HP
- c) AP
- d) None of the mentioned

Answer: c

Explanation: If  $a_1, a_2, \dots$  are in AP then  $a_1^{-1}, a_2^{-1}, \dots$  are in Harmonic Progression.

5. If  $a, b, c$  are in hp, then  $b$  is related with  $a$  and  $c$  as \_\_\_\_\_

- a)  $2(\frac{1}{b}) = (\frac{1}{a} + \frac{1}{c})$
- b)  $2(\frac{1}{c}) = (\frac{1}{b} + \frac{1}{c})$

- c)  $2(\frac{1}{a}) = (\frac{1}{a} + \frac{1}{b})$   
 d) None of the mentioned

Answer: a

Explanation:  $\frac{1}{a}$ ,  $\frac{1}{b}$ ,  $\frac{1}{c}$  will be in arithmetic series and  $\frac{1}{b}$  will be the AM of a, c.

6. For number A, C if H is harmonic mean, G is geometric mean then  $H \geq G$ .

- a) True  
 b) False

Answer: b

Explanation: Geometric mean is always greater than or equal to the harmonic mean.

7. For number B, C if H is harmonic mean, A is the arithmetic mean then  $H \geq A$ .

- a) True  
 b) False

Answer: b

Explanation: Arithmetic mean is always greater than or equal to harmonic mean.

8. Which of the following gives the right inequality for AM, GM, HM?

- a)  $AM \geq HM \geq GM$   
 b)  $GM \geq AM \geq HM$   
 c)  $AM \geq GM \geq HM$   
 d)  $GM \geq HM \geq AM$

Answer: c

Explanation: Arithmetic mean is always greater than or equal to geometric mean, geometric mean is always greater than or equal to harmonic mean.

9. For two number a,b HM between them is given by?

- a)  $(2b+2a)/3b$   
 b)  $2ab/(a+b)$   
 c)  $(a+b)/2ab$   
 d)  $2b/(a+b)$

Answer: b

Explanation: Let c be the hm,  $\frac{2}{c} = \frac{1}{a} + \frac{1}{b}$  (AM property),  $c = 2ab/(a+b)$ .

10. If A, G, H are the AM, GM, HM between a and b respectively then?

- a) A, G, H are in hp  
 b) A, G, H are in gp  
 c) A, G, H are in ap  
 d) None of the mentioned

Answer: b

Explanation:  $A = (a+b)/2$ ,  $G = (ab)^{1/2}$ ,  $H = 2b/(a+b)$ , clearly  $A \times H = G^2$  thus A, G, H are in gp.

### Matrices

1. If the order of the matrix is  $m \times n$ , then how many elements will there be in the matrix?

- a)  $mn$
- b)  $m^2 n^2$
- c)  $mn^2$
- d)  $2mn$

Answer: a

Explanation: The number of elements for a matrix with the order  $m \times n$  is equal to  $mn$ , where  $m$  is the number of rows and  $n$  is the number of columns in the matrix.

5. In a matrix with 9 elements, then the possible ordered pairs are (3, 3), (1, 9), (9, 1).

- a) True
- b) False

Answer: a

Explanation: The given statement is true. The possible orders in which the matrix can be formed are  $3 \times 3, 9 \times 1, 1 \times 9$ . Therefore, the ordered pairs are (3,3), (1,9), (9,1).

6. Which of the following is not a possible ordered pair for a matrix with 6 elements.

- a) (2,3)
- b) (3,2)
- c) (1,6)
- d) (3,1)

Answer: d

Explanation: The possible orders in which the matrix with 6 elements can be formed are  $2 \times 3, 3 \times 2, 1 \times 6, 6 \times 1$ . Therefore, the possible orders pairs are (2,3), (3,2), (1,6), (6,1). Thus, (3,1) is not possible.

1. The matrix which follows the conditions  $m=n$  is called?

- a) Square matrix
- b) Rectangular matrix
- c) Scalar matrix
- d) Diagonal matrix

Answer: a

Explanation: A square matrix is a matrix in which the number of rows( $m$ ) is equal to the number of columns( $n$ ). Therefore, the matrix which follows the condition  $m=n$  is a square matrix.



4. The matrix which follows the condition  $m > n$  is called as \_\_\_\_\_
- a) vertical matrix
  - b) horizontal matrix
  - c) diagonal matrix
  - d) square matrix

Answer: a

Explanation: The matrix in which the number of columns is greater than the number of rows is called a vertical matrix. There the matrix which follows the condition  $m > n$  is a vertical matrix.

9. Which of the following is not a type of matrix?
- a) Scalar matrix
  - b) Diagonal matrix
  - c) Symmetric matrix
  - d) Minor matrix

Answer: d

Explanation: Minor matrix is not a type of matrix. Scalar, diagonal, symmetric are various type of matrices.

1. The addition of matrices is only possible if they are of the same order.
- a) True
  - b) False

Answer: a

Explanation: The given statement is true. Addition of matrices is possible only if the matrices are of the same order. If there are two matrices of different order, then  $A+B$  is not defined.

8. Matrix addition and matrix multiplication both are commutative.
- a) True
  - b) False

Answer: b

Explanation: The given statement is false. Matrix addition is commutative i.e.  $A+B=B+A$ . But matrix multiplication is not commutative i.e.  $AB \neq BA$ .

10. Which of the following conditions is incorrect for matrix multiplication?
- a)  $A(BC)=(AB)C$
  - b)  $A(B+C)=AB+AC$
  - c)  $AB=0$  if either  $A$  or  $B$  is  $0$
  - d)  $AB=BA$

Answer: d

Explanation: Matrix multiplication is never commutative i.e.  $AB \neq BA$ . Therefore, the condition  $AB=BA$  is incorrect.



1. Which of the following is not the property of transpose of a matrix?

- a)  $(A')' = A$
- b)  $(A+B)' = A' + B'$
- c)  $(AB)' = (BA)'$
- d)  $(kA)' = kA'$

Answer: c

Explanation:  $(AB)' = (BA)'$  is incorrect. We know that matrix multiplication is not commutative i.e.  $AB \neq BA$ . Hence, its transpose will also not be commutative.  $(AB)' = B'A'$

9. Which of the following is the reversal law of transposes?

- a)  $(A-B)' = B' - A'$
- b)  $(AB)' = B'A'$
- c)  $(AB)' = (BA)'$
- d)  $(A+B)' = B' + A'$

Answer: b

Explanation: According to the reverse law of transposes the transpose of the product is the product of the transposes taken in the reverse order i.e.  $(AB)' = B'A'$ .

2. Which of the following conditions holds true for a symmetric matrix?

- a)  $A = -A'$
- b)  $A = A'$
- c)  $A = IA$
- d)  $A = |A|$

Answer: b

Explanation: A matrix is said to be a symmetric matrix if it is equal to its transpose i.e.  $A = A'$ .

3. Which of the following conditions holds true for a skew-symmetric matrix?

- a)  $A = IA$
- b)  $A = |A|$
- c)  $A = A'$
- d)  $A = -A'$

Answer: d

Explanation: A matrix is said to be skew-symmetric if it is equal to the negative of its transpose i.e.  $A = -A'$ .

4. Any square matrix can be expressed as a sum of symmetric and skew-symmetric matrix.

- a) True
- b) False

Answer: a

Explanation: The given statement is true. Every square matrix can be expressed as a sum of symmetric and skew-symmetric matrix.

If  $A$  is a square matrix then it can be expressed as

$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$ , where  $(A+A')$  is symmetric and  $(A-A')$  is skew-symmetric.

1. How many elementary operations are possible on Matrices?

- a) 3
- b) 2
- c) 6
- d) 5

Answer: c

Explanation: There are a total of 6 elementary operations that are possible on matrices, three on rows and three on columns.

2. Which of the following is not a valid elementary operation?

- a)  $R_i \leftrightarrow R_j$
- b)  $R_i \rightarrow R_j + kR_i$
- c)  $R_i \rightarrow kR_i$
- d)  $R_i \rightarrow 1 + kR_i$

Answer: d

Explanation: The elementary operation  $R_i \rightarrow 1 + kR_i$  is incorrect, the valid elementary operations on matrices are as follows.

- i) Interchanging any two rows and columns
- ii) The multiplication of the elements of any row or column by a non-zero number.
- iii) The addition to the elements of any row or column, the corresponding elements of any other row and column multiplied by any non-zero number.

3. If A and B are invertible matrices of the same order, then  $(AB)^{-1} = B^{-1} A^{-1}$ .

- a) True
- b) False

Answer: a

4. A matrix A is invertible if it has all zeroes in one or more rows on L.H.S.

- a) True
- b) False

Answer: b

9. Which of the following is not a property of invertible matrices if A and B are matrices of the same order?

- a)  $(AB)^{-1} = A^{-1} B^{-1}$
- b)  $(AA^{-1}) = (A^{-1} A) = I$
- c)  $(AB)^{-1} = B^{-1} A^{-1}$
- d)  $AB = BA = I$

Answer: a

Explanation:  $(AB)^{-1} = A^{-1} B^{-1}$  is incorrect. The correct formula is  $(AB)^{-1} = B^{-1} A^{-1}$ .  $B^{-1} A^{-1} \neq A^{-1} B^{-1}$  as matrix multiplication is not commutative.

### **Determinants**

1. Which of the following is not a property of determinant?

- a) The value of determinant changes if all of its rows and columns are interchanged
- b) The value of determinant changes if any two rows or columns are interchanged
- c) The value of determinant is zero if any two rows and columns are identical
- d) The value of determinant gets multiplied by  $k$ , if each element of row or column is multiplied by  $k$

Answer: a

Explanation: The value of determinant remains unchanged if all of its rows and columns are interchanged i.e.  $|A| = |A'|$ , where  $A$  is a square matrix and  $A'$  is the transpose of the matrix  $A$ .

2. What are the number of solutions of linear equation  $2x+4y=12$ ?

- a) One
- b) Two
- c) Three
- d) Infinite

Answer: d



### **Prime Numbers**

1. The number of factors of prime numbers are \_\_\_\_\_

- a) 2
- b) 3
- c) Depends on the prime number
- d) None of the mentioned

Answer: a

Explanation: A prime number is only divisible by 1 and itself.

2. What is the number '1'?

- a) Prime number
- b) Composite number
- c) Neither Prime nor Composite
- d) None of the mentioned

Answer: c

Explanation: 1 is neither prime number nor composite.

3. All prime numbers are odd.

- a) True
- b) False

Answer: b

Explanation: 2 is even as well as prime.

4. 3 is the smallest prime number possible.

- a) True
- b) False

Answer: b

Explanation: 2 is also a prime number.

5. How many prime numbers are there between 1 to 20?

- a) 5
- b) 6
- c) 7
- d) None of the mentioned

Answer: d

Explanation: The prime numbers between 1 to 20 are 2, 3, 5, 7, 11, 13, 17, 19.

6. There are a finite number of prime numbers.

- a) True
- b) False

Answer: b

Explanation: There are infinite numbers of primes.

7. Sum of two different prime number is a \_\_\_\_\_

- a) Prime number
- b) Composite number
- c) Either Prime or Composite
- d) None of the mentioned

Answer: c

Explanation: Eg:-  $2 + 3 = 5$  a prime,  $3 + 7 = 10$  a composite.

8. Difference of two distinct prime numbers is?

- a) Odd and prime
- b) Even and composite
- c) None of the mentioned
- d) All of the mentioned

Answer: c

Explanation:  $3 - 2 = 1$  is neither prime nor composite.

9. If a, b, c, d are distinct prime numbers with an as smallest prime then  $a * b * c * d$  is a \_\_\_\_\_

- a) Odd number
- b) Even number
- c) Prime number
- d) None of the mentioned

Answer: b

Explanation: Since a is 2,  $2 * b * c * d = \text{Even number}$ .

10. If a, b are two distinct prime number than a highest common factor of a, b is \_\_\_\_\_

- a) 2
- b) 0
- c) 1
- d) ab

Answer: c

Explanation: HCF of two prime numbers is 1.

### **Divisibility**

1. Find the value of X, if 1245X42 is divisible by 11.

- a) 4
- b) 5
- c) 8
- d) 1

Answer: a

Explanation: We know that, for a number to be divisible by 11, the difference between the sum of digits in odd places and the sum of digits in even places should be either 0 or a number divisible by 11.

2. Which of the following is/are true?

- I. For a number to be divisible by 19, multiply last digit by 2 and add it to the rest of the number and the answer should be a multiple of 19.
- II. For a number to be divisible by 29, multiply last digit by 3 and subtract it to the rest of the number and the answer should be a multiple of 29.

- a) Only I
- b) Only II
- c) Both I & II
- d) Neither I nor II

Answer: a

Explanation: The statement I is true i.e., for a number to be divisible by 19, multiply last digit by 2 and add it to the rest of the number and the answer should be a multiple of 19.

The statement II is false because, for a number to be divisible by 29, multiply last digit by 3 and add it to the rest of the number and the answer should be a multiple of 29.

3. If 23XY70 is a number with all distinct digits and divisible by 11, find XY.

- a) 63
- b) 61
- c) 23 or 43
- d) 61 or 17

Answer: b

Explanation: We know that, for a number to be divisible by 11, the difference between the sum of digits in odd places and the sum of digits in even places should be either 0 or a number divisible by 11.

4. Which of the following number is divisible by 19?

- a) 703
- b) 629
- c) 697
- d) 559

Answer: a

Explanation: For a number to be divisible by 19, multiply last digit by 2 and add it to the rest of the number and the answer should be a multiple of 19.  $70 + 2 \times 3 = 76$  which is divisible by 19. Hence, 703 is divisible by 19.

5. Which of the following number divides 7386071?

- a) 4
- b) 9
- c) 3
- d) 11

Answer: d

Explanation: For a number to be divisible by 4, last two digits of the number should be divisible by 4. But 71 is not divisible by 4. Hence the number is not divisible by 4.

For a number to be divisible by 3, the sum of the digits should be divisible by 3. But,  $7+3+8+6+0+7+1=32$  is not divisible by 3. Hence the number is not divisible by 3.

For a number to be divisible by 9, the sum of the digits should be divisible by 9. But,  $7+3+8+6+0+7+1=32$  is not divisible by 9. Hence the number is not divisible by 9.

For a number to be divisible by 11, the difference between the sum of digits in odd places and the sum of digits in even places should be either 0 or a number divisible by 11.  $(7+8+0+1) - (3+6+7) = 0$ , which is divisible by 11. Hence the number is divisible by 11.

6. If a positive integer  $n$  is divisible by 7, the remainder is 3, which of the following number yields remainder 1 when divided by 7?

- a)  $n+3$
- b)  $n-5$

- c)  $n+6$
- d)  $n+5$

Answer: d

Explanation: For example, let  $n=35$ . 35 is divisible by 7. Hence 38 is divisible by 7 with 3 as remainder.

7. Which of the following number is divisible by 7, 11 and 13?

- a) 2002
- b) 2004
- c) 2007
- d) 2011

Answer: a

8. If  $p$  and  $q$  are integers divisible by 5, which of the following is not necessarily true?

- a)  $p + q$  is divisible by 10
- b)  $p - q$  is divisible by 5
- c)  $p^2 - q^2$  is divisible by 25
- d)  $p \cdot q$  is divisible by 25

Answer: a

Explanation: Let  $p = 15$  and  $q = 10$ , which are divisible by 5.

9. If a number is divisible by both 7 and 13, then in which of the number case the number is divisible?

- a) 91
- b) 20
- c) 4
- d) 40

Answer: a

Explanation: We know that 7 and 13 are co-prime. If a number is divisible by both 7 and 13, then it should also, be divisible by  $7 \cdot 13 = 91$ .

10. Which of the following is divisible by 29?

- a) 899
- b) 889
- c) 799
- d) 769

Answer: a

Explanation: For a number to be divisible by 29, multiply last digit by 3 and add it to the rest of the number and the answer should be a multiple of 29.

$89 + 3 \cdot 9 = 116 = 11 + 3 \cdot 6 = 29$ , which is divisible by 29. Hence, 899 is divisible by 29.

1. What least number should be added to 3000 to obtain a number exactly divisible by 17?

- a) 9
- b) 12
- c) 5
- d) 7

Answer: a

Explanation: On dividing 3000 by 17, we get 8 as remainder. Number to be added =  $(17 - 8) = 9$ .

2. Find the least number that must be subtracted from 2000 to get a number exactly divisible by 31.

- a) 6
- b) 16
- c) 21
- d) 30

Answer: b

Explanation: On dividing 2000 by 31, we get 16 as remainder. Required number to be subtracted = 16.

3.  $m$  is a positive integer such that  $m^2 + 12$  is divisible by  $m$ . Find all the possible values of  $m$ .

- a) 1, 2, 4, 6 and 12
- b) 1, 2, 3, 6 and 12
- c) 1, 2, 3, 4, 6 and 12
- d) 1, 2, 3, 4, 8 and 12

Answer: c

4. Find the greatest 5-digit number which is exactly divisible by 47.

- a) 99969
- b) 99981
- c) 99975
- d) 99993

Answer: a

5. A chewing gum produced 58335 packs of gum. The company produced the same number of packs of each flavor of gum. How many different flavors of gum could the company have produced?

- a) 6
- b) 8
- c) 7
- d) 3

Answer: b

6. Which of the following numbers divide 11111111?

- a) 3 and 37 only
- b) 3, 37 and 111 only





- c) 3, 11, 37 and 111 only
- d) 3, 11, 37, 111 and 1001

Answer: b

7. Which of the following is not necessarily true for a number  $m(m+1)(2m+1)$ ,  $m$  being a natural number?

- a) It is divisible by 3
- b) It is always even
- c) It is never divisible by 237
- d) It is divisible by the sum of the square of first  $n$  natural number

Answer: c

8. What is the value of  $n$  when 34041 and 32506 are divided by a 3-digit number  $n$ , leave the same remainder?

- a) 307
- b) 461
- c) 298
- d) 359

Answer: a

9. Find the number of positive integer  $p$  in the range  $50 \geq p \geq 10$ , such that the product  $(p-1)(p-2)(p-3) \dots 3 \cdot 2 \cdot 1$  is not divisible by  $p$ .

- a) 9
- b) 11
- c) 13
- d) 8

Answer: b

10. If the sum of numbers  $(x+4)^2$  and  $x^3$  is divisible by 7, then which of the following may be a least value of  $x$ ?

- a) 5
- b) 3
- c) 7
- d) 6

Answer: b

### **GCD & LCM**

1. Which of the following is not another name for GCD(Greatest Common Divisor)?

- a) LCM
- b) GCM

- c) GCF
- d) HCF

Answer: a

Explanation: : LCM (Least Common Multiple) and GCD are not same. GCM (Greatest Common Measure), GCF (Greatest Common Factor), HCF (Highest Common Factor) are other names for GCD.

2. What is the GCD of 8 and 12?

- a) 8
- b) 12
- c) 2
- d) 4

Answer: d

Explanation: GCD is largest positive integer that divides each of the integer. So the GCD of 8 and 12 is 4.

3. If GCD of two number is 8 and LCM is 144, then what is the second number if first number is 72?

- a) 24
- b) 2
- c) 3
- d) 16

Answer: d

Explanation: As  $A * B = \text{GCD}(A, B) * \text{LCM}(A, B)$ . So  $B = (144 * 8)/72 = 16$ .

4. Which of the following is also known as GCD?

- a) Highest Common Divisor
- b) Highest Common Multiple
- c) Highest Common Measure
- d) Lowest Common Multiple

Answer: a

Explanation: GCM (Greatest Common Measure), GCF (Greatest Common Factor), HCF (Highest Common Factor) and HCF (Highest Common Divisor) are also known as GCD.

5. Which of the following is coprime number?

- a) 54 and 24
- b) 4 and 8
- c) 6 and 12
- d) 9 and 28

Answer: d

Explanation: Coprime numbers have GCD 1. So 9 and 28 are coprime numbers. While 54 and 24 have GCD 6, 4 and 8 have GCD 4, 6 and 12 have GCD 6.

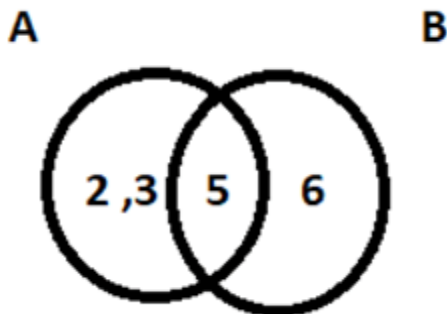
6. In terms of Venn Diagram, which of the following expression gives GCD (Given  $A \cap B \neq \emptyset$ )?

- a) Multiplication of  $A \cup B$  terms
- b) Multiplication of  $A \cap B$  terms
- c) Multiplication of  $A \cdot B$  terms
- d) Multiplication of  $A - B$  terms

Answer: b

Explanation: In terms of Venn Diagram, the GCD is given by the intersection of two sets. So  $A \cap B$  gives the GCD. While  $A \cup B$  gives the LCM.

7. What is the GCD according to the given Venn Diagram?



- a) 2
- b) 3
- c) 5
- d) 6

Answer: c

Explanation: In terms of Venn Diagram, the GCD is given by the intersection of two sets. So  $A \cap B$  gives the GCD. While  $A \cup B$  gives the LCM. So here  $A \cap B$  is 5.

8. What is the GCD of a and b?

- a)  $a + b$
- b)  $\gcd(a-b, b)$  if  $a > b$
- c)  $\gcd(a+b, a-b)$
- d)  $a - b$

Answer: b

Explanation: As per Euclid's Algorithm,  $\gcd(a, b) = \gcd(a-b, b)$  if  $a > b$  or  $\gcd(a, b) = \gcd(a, b-a)$  if  $b > a$ .

9. What is the GCD of 48, 18, 0?

- a) 24
- b) 2
- c) 3
- d) 6

Answer: d

Explanation: GCD is largest positive integer that divides each of the integer. So the GCD of 48, 18, 0 is 6.

10. Is 9 and 28 coprime number?

- a) True
- b) False

Answer: a

Explanation: Coprime numbers have GCD 1. So 9 and 28 are coprime numbers.

11. If  $\gcd(a, b)$  is defined by the expression,  $d = a \cdot p + b \cdot q$  where  $d, p, q$  are positive integers and  $a, b$  is both not zero, then what is the expression called?

- a) Bezout's Identity
- b) Multiplicative Identity
- c) Sum of Product
- d) Product of Sum

Answer: a

Explanation: If  $\gcd(a, b)$  is defined by the expression,  $d = a \cdot p + b \cdot q$  where  $d, p, q$  are positive integers and  $a, b$  is both not zero, then the expression is called Bezout's Identity and  $p, q$  can be calculated by extended form of Euclidean algorithm.

12. Is gcd an associative function.

- a) True
- b) False

Answer: a

Explanation: The gcd function is an associative function as  $\gcd(a, \gcd(b, c)) = \gcd(\gcd(a, b), c)$ .

13. Which is the correct term of the given relation,  $\gcd(a, b) \cdot \text{lcm}(a, b) = ?$

- a)  $|a \cdot b|$
- b)  $a + b$
- c)  $a - b$
- d)  $a / b$

Answer: a

Explanation: The gcd is closely related to lcm as  $\gcd(a, b) \cdot \text{lcm}(a, b) = |a \cdot b|$ .

14. Who gave the expression for the probability and expected value of gcd?

- a) James E. Nymann
- b) Riemann
- c) Thomae
- d) Euler



Answer: a

Explanation: In the year 1972, James E. Nymann showed some result to show the probability and expected value of gcd.

15. What is the computational complexity of Binary GCD algorithm where a and b are integers?

- a)  $O((\log a + \log b)^2)$
- b)  $O(\log(a + b))$
- c)  $O(\log ab)$
- d)  $O(\log a - b)$

Answer: a

Explanation: From the Binary GCD algorithm, it is found that the computational complexity is  $O((\log a + \log b)^2)$  as the total number of steps in the execution is at most the total sum of number of bits of a and b.

1. LCM is also called as \_\_\_\_\_

- a) GCD
- b) SCM
- c) GCF
- d) HCF

Answer: b

Explanation: GCD (Greatest Common Divisor), GCF (Greatest Common Factor), HCF (Highest Common Factor) is not an alias for LCM. LCM is also called as Smallest Common Multiple(SCM).

2. What is the LCM of 8 and 13?

- a) 8
- b) 12
- c) 20
- d) 104

Answer: d

Explanation: 104 is the smallest positive integer that is divisible by both 8 and 12.

3. Which is the smallest number of 3 digits that is divisible by 2, 4, 8?

- a) 100
- b) 102
- c) 116
- d) 104

Answer: d

Explanation: LCM of 2, 4, 8 is 8. So check for the number that is divisible by 8. So 104 is the smallest number that is divisible by 2, 4, 8.

4. Which of the following is also known as LCM?

- a) Lowest Common Divisor
- b) Least Common Multiple

- c) Lowest Common Measure
- d) Highest Common Multiple

Answer: a

Explanation: Least Common Multiple is also known as LCM or Lowest Common Multiple.

5. What is the LCM of two coprime numbers?

- a) 1
- b) 0
- c) Addition of two coprime numbers
- d) Multiplication of two coprime numbers

Answer: d

Explanation: Coprime numbers have GCD 1. While LCM of coprime numbers is the product of those two coprime numbers.

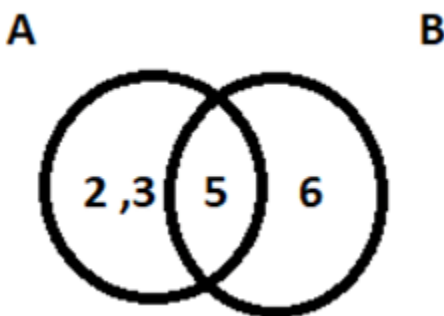
6. In terms of Venn Diagram, which of the following expression gives LCM (Given  $A \cap B \neq \emptyset$ )?

- a) Multiplication of  $A \cup B$  terms
- b) Multiplication of  $A \cap B$  terms
- c) Multiplication of  $A \cdot B$  terms
- d) Multiplication of  $A - B$  terms

Answer: a

Explanation: In terms of Venn Diagram, the LCM is given by the Union of two sets. So  $A \cup B$  gives the LCM. While  $A \cap B$  gives the GCD.

7. What is the LCM according to the given Venn Diagram?



- a) 2
- b) 3
- c) 180
- d) 6

Answer: c

Explanation: In terms of Venn Diagram, the LCM is given by the Union of two sets. So  $A \cup B$  gives the LCM. So product of all the terms is 180.

8. What is the lcm (a, b)?

- a)  $a + b$
- b)  $\gcd(a-b, b)$  if  $a > b$
- c)  $\text{lcm}(b, a)$
- d)  $a - b$

Answer: c

Explanation: Since the LCM function is commutative, so  $\text{lcm}(a, b) = \text{lcm}(b, a)$ .

9. What is the LCM of 48, 18, 6?

- a)  $12^2$
- b)  $12 \times 2$
- c) 3
- d) 6

Answer: a

Explanation: The LCM of 48, 18, 6 is 144 and  $12^2$  is 144.

10. Is 9 and 28 coprime number.

- a) True
- b) False

Answer: a

Explanation: Coprime numbers have GCD 1 and LCM is the product of the two given terms. So 9 and 28 are coprime numbers.

11. What is the following expression,  $\text{lcm}(a, \text{lcm}(b, c))$  equal to?

- a)  $\text{lcm}(a, b, c)$
- b)  $a \times b \times c$
- c)  $a + b + c$
- d)  $\text{lcm}(\text{lcm}(a, b), c)$

Answer: d

Explanation: Since LCM function follows associativity, hence  $\text{lcm}(a, \text{lcm}(b, c))$  is equal to  $\text{lcm}(\text{lcm}(a, b), c)$ .

12. Is lcm an associative function.

- a) True
- b) False

Answer: a

Explanation: The lcm function is an associative function as  $\text{lcm}(a, \text{lcm}(b, c))$  is equal to  $\text{lcm}(\text{lcm}(a, b), c)$ .

13. Which is the correct term of the given relation,  $\text{lcm}(a, b) \times \gcd(a, b) = ?$

- a)  $|a \times b|$
- b)  $a + b$

- c)  $a - b$
- d)  $a / b$

Answer: a

Explanation: The lcm is closely related to gcd as  $\text{lcm}(a, b) * \text{gcd}(a, b) = |a*b|$ .

14. What is the following expression,  $\text{lcm}(a, \text{gcd}(a, b))$  equal to?

- a) a
- b) b
- c)  $a*b$
- d)  $a + b$

Answer: a

Explanation: Since the lcm function follows absorption laws so  $\text{lcm}(a, \text{gcd}(a, b))$  equal to a.

15. Which algorithm is the most efficient numerical algorithm to obtain lcm?

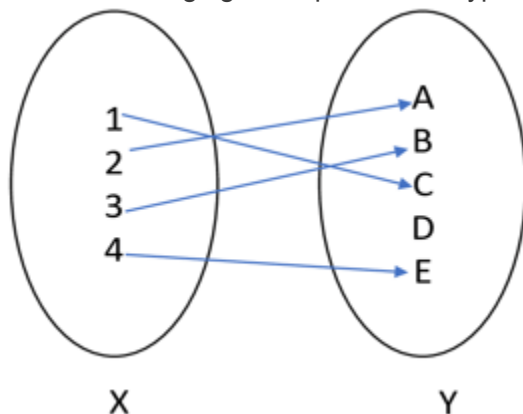
- a) Euler's Algorithm
- b) Euclid's Algorithm
- c) Chebyshev Function
- d) Partial Division Algorithm

Answer: b

Explanation: The most efficient way of calculating the lcm of a given number is using Euclid's algorithm which computes the lcm in much lesser time compared to other algorithms.

## Functions

1. The following figure depicts which type of function?



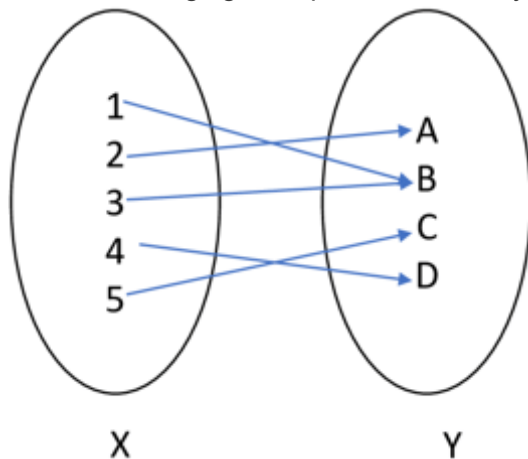
- a) one-one
- b) onto
- c) many-one
- d) both one-one and onto

Answer: a





2. The following figure represents which type of function?



- a) one-one
- b) onto
- c) many-one
- d) neither one-one nor onto

Answer: b

Explanation: The above function is onto or surjective. A function  $f: X \rightarrow Y$  is said to be surjective or onto if, every element of Y is the image of some elements in X. The condition for a surjective function is for every  $y \in Y$ , there is an element in X such that  $f(x) = y$ .

3. A function  $f: \mathbb{N} \rightarrow \mathbb{N}$  is defined by  $f(x) = x^2 + 12$ . What is the type of function here?

- a) bijective
- b) surjective
- c) injective
- d) neither surjective nor injective

Answer: c

Explanation: The above function is an injective or one-one function.

4. A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = 5x^3 - 8$ . The type of function is \_\_\_\_\_

- a) one -one
- b) onto
- c) many-one
- d) both one-one and onto

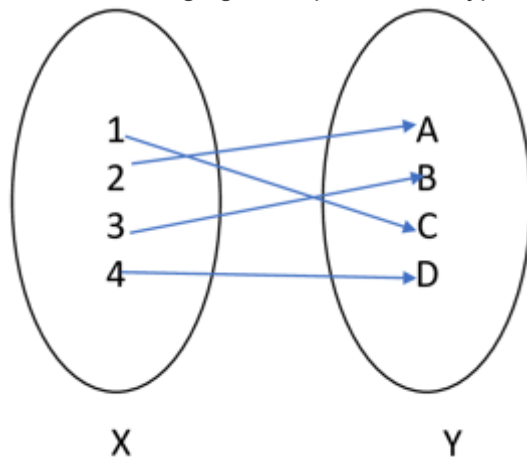
Answer: c

5. The function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined as  $f(x) = 7x + 4$  is both one-one and onto.

- a) True
- b) False

Answer: a

6. The following figure depicts which type of function?



- a) injective
- b) bijective
- c) surjective
- d) neither injective nor surjective

Answer: b

7. A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 5x^4 + 2$  is one – one but not onto.

- a) True
- b) False

Answer: b

8. Let  $A = \{1, 2, 3\}$  and  $B = \{4, 5, 6\}$ . Which one of the following functions is bijective?

- a)  $f = \{(2, 4), (2, 5), (2, 6)\}$
- b)  $f = \{(1, 5), (2, 4), (3, 4)\}$
- c)  $f = \{(1, 4), (1, 5), (1, 6)\}$
- d)  $f = \{(1, 4), (2, 5), (3, 6)\}$

Answer: d

9. Let  $P = \{10, 20, 30\}$  and  $Q = \{5, 10, 15, 20\}$ . Which one of the following functions is one – one and not onto?

- a)  $f = \{(10, 5), (10, 10), (10, 15), (10, 20)\}$
- b)  $f = \{(10, 5), (20, 10), (30, 15)\}$
- c)  $f = \{(20, 5), (20, 10), (30, 10)\}$
- d)  $f = \{(10, 5), (10, 10), (20, 15), (30, 20)\}$

Answer: b

10. Let  $M = \{5, 6, 7, 8\}$  and  $N = \{3, 4, 9, 10\}$ . Which one of the following functions is neither one-one nor onto?

- a)  $f=\{(5,3),(5,4),(6,4),(8,9)\}$
- b)  $f=\{(5,3),(6,4),(7,9),(8,10)\}$
- c)  $f=\{(5,4),(5,9),(6,3),(7,10),(8,10)\}$
- d)  $f=\{(6,4),(7,3),(7,9),(8,10)\}$

Answer: a

### **Sets & Relations**

1. How to define a set?

- a) A collection of well-defined objects or element
- b) A collection of unordered objects or element
- c) Any random elements
- d) A collection of special characters

Answer: a

2. How is a set denoted?

- a) ()
- b) {}
- c) []
- d) \*\*

Answer: b

Explanation: A set is represented by  $\{ \}$ . Usually, but not necessarily a set is denoted by a capital letter e.g. A, B,..... V, W, X, Y, Z. The elements are enclosed between  $\{ \}$  denoted by small letters a, b, ....., y, z.

3. How will you define a set of all real numbers?

- a)  $\{x: -1 < x < 1\}$
- b)  $\{x: 0 < x < \infty\}$
- c)  $\{x: -\infty < x < \infty\}$
- d)  $\{x: -Z < x < +Z\}$



Answer: c

Explanation: All the numbers whether it is an integer or rational number or irrational number is defined as Real Number. The range of the real number lies between in the range  $(-\infty, +\infty)$ .

4. How will you define Union of two sets A and B?

- a)  $\{x: x \in A \text{ or } x \in B\}$
- b)  $\{x: x \in A \text{ or } x \in B \text{ (or both)}\}$
- c)  $\{x: x \in A \text{ and } B\}$
- d)  $\{x: x \in A - B\}$

Answer: b

5. How will you define the difference of two sets B-A?

- a)  $\{x: x \in A \text{ and } x \notin B\}$
- b)  $\{x: x \notin A \text{ and } x \in B\}$
- c)  $\{x: x \in A \text{ and } x \in B\}$
- d)  $\{x: x \notin A \text{ and } x \notin B\}$

Answer: b

Explanation: The difference of a set A and B is denoted as A-B. A-B is a set of those elements that are in the set A but not in the set B. Similarly, the difference of a set B and A is denoted as B-A. It is a set of those elements that are in the set B but not in the set A.

6. What will be the set of the interval (a, b]?

- a)  $\{x: a < x < b\}$
- b)  $\{x: a \leq x \leq b\}$
- c)  $\{x: a < x \leq b\}$
- d)  $\{x: a \leq x < b\}$

Answer: c

Explanation: The symbol ( ) implies that the value will always be less than or greater than the x value i.e. end points are not included.

{ } implies that all the values that does not satisfy a given interval are included inside {}.

[ ] implies that the value will always be less than equal to or greater than equal to the x value i.e. end points are included. This is possible only when both a and b are finite.

7. How to define Wavy Curve Method f(x)?

- a)  $(x-a_1)^{n_1} / (x-a_2)^{n_2} / (x-a_3)^{n_3} \dots \dots \dots / (x-a_k)^{n_k} * (x-b_1)^{m_1} / (x-b_2)^{m_2} / (x-b_3)^{m_3} \dots \dots \dots / (x-b_p)^{m_p}$
- b)  $(x-a_1)^{n_1} + (x-a_2)^{n_2} + (x-a_3)^{n_3} \dots \dots \dots + (x-a_k)^{n_k} / (x-b_1)^{m_1} + (x-b_2)^{m_2} + (x-b_3)^{m_3} \dots \dots \dots + (x-b_p)^{m_p}$
- c)  $(x-a_1)^{n_1} (x-a_2)^{n_2} (x-a_3)^{n_3} \dots \dots \dots (x-a_k)^{n_k} / (x-b_1)^{m_1} (x-b_2)^{m_2} (x-b_3)^{m_3} \dots \dots \dots (x-b_p)^{m_p}$
- d)  $(x-a_1)^{n_1} - (x-a_2)^{n_2} - (x-a_3)^{n_3} \dots \dots \dots - (x-a_k)^{n_k} / (x-b_1)^{m_1} - (x-b_2)^{m_2} - (x-b_3)^{m_3} \dots \dots \dots - (x-b_p)^{m_p}$

Answer: c

8. How to solve for x, if  $|x-1| \geq 3$ ?

- a)  $(-\infty, -2) \cup (4, \infty)$
- b)  $(-\infty, -2] \cup [4, \infty)$
- c)  $(0, -2] \cup (4, 0)$
- d)  $(-\infty, \infty) - \{-2, 4\}$

Answer: b

Explanation: Given,  $|x-1| \geq 3 = x-1 < -3$  or  $x-1 \geq 3 = x \leq -2$  or  $x \geq 4$ . Hence,  $x \in (-\infty, -2] \cup [4, \infty)$ .

9. What is the interval of  $f(x) = (x-1)(x-2)(x-3)/(x^3 + 6x^2 + 11x + 6)$  where f(x) is positive?

- a)  $(-\infty, -3) \cup (3, \infty)$
- b)  $(3, -2) \cup (1, 1) \cup (2, 3)$

- c)  $(-\infty, -3) \cup (2, -1) \cup (1, 2) \cup (3, \infty)$   
d)  $(-\infty, \infty)$

Answer: c

10. Which of the following is not a set of letters of word PRINCIPAL?

- a) {P,R,I,N,C,A,L}  
b) {C,A,P,I,N,R,L}  
c) {P,R,I,N,C,I,P,A,L}  
d) {L,N,I,P,C,A,R}

Answer: c

Explanation: A set has all unique elements. So the set which contain all the elements of word PRINCIPAL and no letter is repeated. Hence, {P,R,I,N,C,I,P,A,L} cannot be a set.

11. A set can be a collection but a collection cannot be a set.

- a) True  
b) False

Answer: a

Explanation: A collection becomes a set when it is well defined for example a collection of good football players is not a set since the phrase "good football players" is vague and not defined.

12. Write the set  $\{x : x \text{ is a natural number and } x^2 - 9 = 0\}$  in roster form.

- a) {3}  
b) {-3}  
c) {3,-3}  
d) {9,3}

Answer: a

Explanation: Since x is given as natural number so x can be positive only.  $x^2 - 9 = 0 \Rightarrow (x-3)(x+3) = 0 \Rightarrow x = 3, -3$ . Here, -3 is not a natural number so, the set  $\{x : x \text{ is a natural number and } x^2 - 9 = 0\}$  can be written as {3}.

13. Let  $X = \{1, 2, 3, 4, 5, 6\}$ . Insert appropriate symbol in  $9 \text{ _____ } X$ .

- a) =  
b) <  
c)  $\in$   
d)  $\notin$

Answer: d

Explanation: Here, 9 is not an element of set X. So, 9 does not belongs to set A.  $9 \notin X$ .

14. Which of the following does not belong to set  $\{x : x \text{ is a vowel in English alphabet}\}$ ?

- a) e  
b) b

- c) i
- d) o

Answer: b

Explanation: There are five vowels in English Alphabet a, e, i, o, u. So, set can be written in roster form as {a, e, i, o, u}. 'b' does not belongs to given set.

15. The number of elements in set  $\{x : x \text{ is a letter of word TRIGONOMETRY}\}$  is \_\_\_\_\_
- a) 8
  - b) 7
  - c) 9
  - d) 10

Answer: c

Explanation: The above set can be represented as {T,R,I,G,O,N,M,E,Y}. So, the number of elements in the set is 9.

16. What is the solution set of the equation  $X^2+3X+2=0$  in roster form?
- a)  $\{-1, 2\}$
  - b)  $\{-1, -2\}$
  - c)  $\{1, -2\}$
  - d)  $\{1, 2\}$

Answer: b

17. Which one of the following is the correct representation for the set  $\{x: x \text{ is a positive integer and } x^3 < 50\}$  in roster form?
- a)  $\{0, 1, 2, 3, 4, 5\}$
  - b)  $\{-1, 1, 2, 3\}$
  - c)  $\{1, 2, 3\}$
  - d)  $\{0, 1, 2, 3\}$

Answer: c

Explanation: 0 is not a positive integer moreover  $3^3 < 50$  and  $4^3 > 50$

18. Which one of the following is not a set?
- a) The collection of all whole numbers less than 200
  - b) The collection of all boys in your class
  - c) The collection of talented actors in Hollywood
  - d) The collection of all books written by Chetan Bhagat

Answer: c

Explanation: The collection of actors is not a set as there is no specific criterion to determine whether an actor is talented or not.

### **Relation**

1. Which of these is not a type of relation?

- a) Reflexive
- b) Surjective
- c) Symmetric
- d) Transitive

Answer: b

Explanation: Surjective is not a type of relation. It is a type of function. Reflexive, Symmetric and Transitive are type of relations.

2. An Equivalence relation is always symmetric.

- a) True
- b) False

Answer: a

Explanation: The given statement is true. A relation  $R$  in a set  $A$  is said to be an equivalence relation if  $R$  is reflexive, symmetric and transitive. Hence, an equivalence relation is always symmetric.

3. Which of the following relations is symmetric but neither reflexive nor transitive for a set  $A = \{1, 2, 3\}$ .

- a)  $R = \{(1, 2), (1, 3), (1, 4)\}$
- b)  $R = \{(1, 2), (2, 1)\}$
- c)  $R = \{(1, 1), (2, 2), (3, 3)\}$
- d)  $R = \{(1, 1), (1, 2), (2, 3)\}$

Answer: b

Explanation: A relation in a set  $A$  is said to be symmetric if  $(a_1, a_2) \in R$  implies that  $(a_2, a_1) \in R$ , for every  $a_1, a_2 \in R$ .

4. Which of the following relations is transitive but not reflexive for the set  $S = \{3, 4, 6\}$ ?

- a)  $R = \{(3, 4), (4, 6), (3, 6)\}$
- b)  $R = \{(1, 2), (1, 3), (1, 4)\}$
- c)  $R = \{(3, 3), (4, 4), (6, 6)\}$
- d)  $R = \{(3, 4), (4, 3)\}$

Answer: a

Explanation: For the above given set  $S = \{3, 4, 6\}$ ,  $R = \{(3, 4), (4, 6), (3, 6)\}$  is transitive as  $(3, 4) \in R$  and  $(4, 6) \in R$  and  $(3, 6)$  also belongs to  $R$ . It is not a reflexive relation as it does not satisfy the condition  $(a, a) \in R$ , for every  $a \in A$  for a relation  $R$  in the set  $A$ .

5. Let  $R$  be a relation in the set  $N$  given by  $R = \{(a, b) : a + b = 5, b > 1\}$ . Which of the following will satisfy the given relation?

- a)  $(2, 3) \in R$
- b)  $(4, 2) \in R$
- c)  $(2, 1) \in R$
- d)  $(5, 0) \in R$



Answer: a

6. Which of the following relations is reflexive but not transitive for the set  $T = \{7, 8, 9\}$ ?

- a)  $R = \{(7, 7), (8, 8), (9, 9)\}$
- b)  $R = \{(7, 8), (8, 7), (8, 9)\}$
- c)  $R = \{0\}$
- d)  $R = \{(7, 8), (8, 8), (8, 9)\}$

Answer: a

Explanation: The relation  $R = \{(7, 7), (8, 8), (9, 9)\}$  is reflexive as every element is related to itself i.e.  $(a, a) \in R$ , for every  $a \in A$ . and it is not transitive as it does not satisfy the condition that for a relation  $R$  in a set  $A$  if  $(a_1, a_2) \in R$  and  $(a_2, a_3) \in R$  implies that  $(a_1, a_3) \in R$  for every  $a_1, a_2, a_3 \in R$ .

7. Let  $I$  be a set of all lines in a  $XY$  plane and  $R$  be a relation in  $I$  defined as  $R = \{(I_1, I_2) : I_1 \text{ is parallel to } I_2\}$ . What is the type of given relation?

- a) Reflexive relation
- b) Transitive relation
- c) Symmetric relation
- d) Equivalence relation

Answer: d

Explanation: This is an equivalence relation. A relation  $R$  is said to be an equivalence relation when it is reflexive, transitive and symmetric.

Reflexive: We know that a line is always parallel to itself. This implies that  $I_1$  is parallel to  $I_1$  i.e.  $(I_1, I_2) \in R$ . Hence, it is a reflexive relation.

Symmetric: Now if a line  $I_1 \parallel I_2$  then the line  $I_2 \parallel I_1$ . Therefore,  $(I_1, I_2) \in R$  implies that  $(I_2, I_1) \in R$ . Hence, it is a symmetric relation.

Transitive: If two lines  $(I_1, I_3)$  are parallel to a third line  $(I_2)$  then they will be parallel to each other i.e. if  $(I_1, I_2) \in R$  and  $(I_2, I_3) \in R$  implies that  $(I_1, I_3) \in R$ .

8. Which of the following relations is symmetric and transitive but not reflexive for the set  $I = \{4, 5\}$ ?

- a)  $R = \{(4, 4), (5, 4), (5, 5)\}$
- b)  $R = \{(4, 4), (5, 5)\}$
- c)  $R = \{(4, 5), (5, 4)\}$
- d)  $R = \{(4, 5), (5, 4), (4, 4)\}$

Answer: d

Explanation:  $R = \{(4, 5), (5, 4), (4, 4)\}$  is symmetric since  $(4, 5)$  and  $(5, 4)$  are converse of each other thus satisfying the condition for a symmetric relation and it is transitive as  $(4, 5) \in R$  and  $(5, 4) \in R$  implies that  $(4, 4) \in R$ . It is not reflexive as every element in the set  $I$  is not related to itself.

9.  $(a, a) \in R$ , for every  $a \in A$ . This condition is for which of the following relations?

- a) Reflexive relation



- b) Symmetric relation
- c) Equivalence relation
- d) Transitive relation

Answer: a

Explanation: The above is the condition for a reflexive relation. A relation is said to be reflexive if every element in the set is related to itself.

10.  $(a_1, a_2) \in R$  implies that  $(a_2, a_1) \in R$ , for all  $a_1, a_2 \in A$ . This condition is for which of the following relations?

- a) Equivalence relation
- b) Reflexive relation
- c) Symmetric relation
- d) Universal relation

Answer: c

### **Combination & Permutation**

1. If  $16Pr-1 : 15Pr-1 = 16 : 7$  then find r.

- a) 10
- b) 12
- c) 7
- d) 8

Answer: a



2. In a colony, there are 55 members. Every member posts a greeting card to all the members. How many greeting cards were posted by them?

- a) 990
- b) 890
- c) 2970
- d) 1980

Answer: c

Explanation: First player can post greeting cards to the remaining 54 players in 54 ways. Second player can post greeting card to the 54 players. Similarly, it happens with the rest of the players. The total numbers of greeting cards posted are

$54 + 54 + 54 \dots$

$54 (55 \text{ times}) = 54 \times 55 = 2970.$

3. Find the number of ways of arranging the letters of the words DANGER, so that no vowel occupies odd place.

- a) 36
- b) 48
- c) 144
- d) 96

Answer: c

Explanation: The given word is DANGER. Number of letters is 6. Number of vowels is 2 (i.e., A, E). Number of consonants is 4 (i.e., D, N, G, R). As the vowels cannot occupy odd places, they can be

arranged in even places. Two vowels can be arranged in 3 even places in  ${}^3P_2$  ways i.e., 6. Rest of the consonants can arrange in the remaining 4 places in  $4!$  ways. The total number of arrangements is  $6 \times 4! = 144$ .

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4. Find the sum of all four digit numbers that can be formed by the digits 1, 3, 5, 7, 9 without repetition.

- a) 666700
- b) 666600
- c) 678860
- d) 665500

Answer: b

Explanation: The given digits are 1, 3, 5, 7, 9

Sum of r digit number =  $n-1P_{r-1}$

(Sum of all n digits)  $\times$  (1111... r times)

N is the number of non zero digits.

Here  $n=5$ ,  $r=4$

The sum of 4 digit numbers

$$4P_3 (1+3+5+7+9)(1111) = 666600.$$

5. If  $nPr = 3024$  and  $nCr = 126$  then find n and r.

- a) 9, 4
- b) 10, 3
- c) 12, 4
- d) 11, 4

Answer: a

Explanation:  $nPr \cdot nCr = 3024 \cdot 126$

$$nPr = n!(n-r)!$$

$$nCr = \frac{n!}{(n-r)! \times r!}$$

$$\text{Hence } [n!(n-r)!] \div [n!(n-r)! \times r!] = 24$$

$$24 = r!$$

$$\text{Hence } r = 4$$

$$\text{Now } nP_4 = 3024$$

$$n!(n-4)! = 3024$$

$$n(n-1)(n-2)(n-3) = 9 \cdot 8 \cdot 7 \cdot 6$$

$$n = 9.$$

6. Find the number of rectangles and squares in an 8 by 8 chess board respectively.

- a) 296, 204
- b) 1092, 204
- c) 204, 1092
- d) 204, 1296

Answer: b

Explanation: Chess board consists of 9 horizontal 9 vertical lines. A rectangle can be formed by any two horizontal and two vertical lines. Number of rectangles =  ${}^9C_2 \times {}^9C_2 = 1296$ . For squares there is one 8 by 8 square four 7 by 7 squares, nine 6 by 6 squares and like this

$$\text{Number of squares on chess board} = 12 + 22 + \dots + 82 = 204$$

$$\text{Only rectangles} = 1296 - 204 = 1092.$$

7. There are 20 points in a plane, how many triangles can be formed by these points if 5 are collinear?

- a) 1130
- b) 550
- c) 1129
- d) 1140

Answer: a

Explanation: Number of points in plane  $n = 20$ .

Number of collinear points  $m = 5$ .

Number of triangles from by joining  $n$  points of which  $m$  are collinear  $= nC_3 - mC_3$

Therefore the number of triangles  $= 20C_3 - 5C_3 = 1140 - 10 = 1130$ .

8. In how many ways can we select 6 people out of 10, of which a particular person is not included?

- a)  $10C_3$
- b)  $9C_5$
- c)  $9C_6$
- d)  $9C_4$

Answer: c

Explanation: One particular person is not included we have to select 6 persons out of 9 which can be done in  $9C_6$  ways.

9. Number of circular permutations of different things taken all at a time is  $n!$ .

- a) True
- b) False

Answer: b

Explanation: The number of circular permutations of different things taken all at a time is  $(n-1)!$  and the number of linear permutations of different things taken all at a time is  $n!$ .

10. Is the given statement true or false?

$$nC_r = nC_{n-r}$$

- a) True
- b) False

Answer: a

Explanation: The property of combination states  $nC_r = nC_{n-r}$

$$\text{As } nC_r = \frac{n!}{r!(n-r)!} \times r!$$

$$nC_{n-r} = \frac{n!}{r!(n-r)!} \times (n-r)! = nC_r.$$

1. Permutation is also known as selection.

- a) True
- b) False

Answer: b

Explanation: Permutation is known as arrangement. Selection is another name for combinations. It involves arrangement of letters, numbers, persons etc.

2.  $nPr =$  \_\_\_\_\_

- a)  $n!$
- b)  $n!r!$

- c)  $n!(n-r)!$   
d)  $n!(n-r)!r!$

Answer: c

Explanation: Permutation is known as arrangement.  $nPr$  means arranging  $r$  objects out of  $n$ .  
 $nPr = \frac{n!}{(n-r)!}$ .

3.  $6! =$  \_\_\_\_\_

- a) 24  
b) 120  
c) 720  
d) 8

Answer: c

Explanation: We know,  $n! = n.(n-1).(n-2).(n-3).....$   
 $6! = 6.5.4.3.2.1 = 720$ .

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4.  $7!5! =$  \_\_\_\_\_

- a) 7  
b) 42  
c) 230  
d) 30

Answer: b

Explanation: We know,  $n! = n.(n-1).(n-2).(n-3)..... = n(n-1)!$   
 $7!5! = 7.6!5! = 7.6.5!5! = 7.6 = 42$ .

5.  $10010! = 18! + x9!$ . Find  $x$ .

- a) 1  
b) 2  
c) 3  
d) 4

Answer: a

Explanation:  $10010! = 18! + x9!$ .

We know,  $n! = n.(n-1).(n-2).(n-3)..... = n(n-1)!$

$10010.9.8! = 18! + x9.8!$

$\Rightarrow 10010.9 = 18! + x9.8!$

$\Rightarrow 109 = 1 + x9$

$\Rightarrow x9 = 109 - 1 = 108$

$\Rightarrow x = 12$ .

6.  $nP_0 =$  \_\_\_\_\_

- a)  $n!$   
b) 1  
c)  $1(n)!$   
d)  $(n-1)!$

Answer: b

Explanation: We know,  $nPr = \frac{n!}{(n-r)!}$ .

$nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$ .



7.  $nP_n =$  \_\_\_\_\_

- a)  $n!$
- b) 1
- c)  $1(n)!$
- d)  $(n-1)!$

Answer: a

Explanation: We know,  $nPr = n!(n-r)!$ .

$$nP_n = n!(n-n)! = n!(0)! = n!$$

8. The number of permutations of  $n$  different objects taken  $r$  at a time, where repetition is allowed is

- \_\_\_\_\_
- a)  $n!$
  - b)  $r!$
  - c)  $nPr$
  - d)  $nr$

Answer: d

Explanation:

The number of permutations of  $n$  different objects taken  $r$  at a time, where repetition is allowed is  $n \times n \times n \times \dots \times n$   $r$  times  $= nr$ .

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9. Find the number of permutations of word DEPENDENT.

- a) 13240
- b) 15120
- c) 16620
- d) 17230

Answer: b

Explanation: There are total 9 letters out of which 1T, 2N, 2D, 3E, 1P.

Total number of permutations are  $9!3!2!2! = 9.8.7.6.5.4.3.2.16 \times 2 \times 2 = 36288024 = 15120$ .

10. Find the number of 5 letter words which can be formed from word IMAGE without repetition using permutations.

- a) 20
- b) 60
- c) 120
- d) 240

Answer: c

Explanation: IMAGE is a 5 letters word. We have to arrange all 5 letters of the word IMAGE without repetition. So, total permutations are  $nPr = 5P_5 = 5! = 5.4.3.2.1 = 120$ .

11. Find the number of 5 letter words that can be formed from word IMAGE using permutations if repetition is allowed.

- a) 25
- b) 120
- c) 125
- d) 3125

Answer: d

Explanation: IMAGE is a 5 letters word. We have to arrange all 5 letters of the word IMAGE with repetition allowed. So, total permutations are  $n^r = 5^5 = 3125$ .

### **Pigeonhole principle**

1. A drawer contains 12 red and 12 blue socks, all unmatched. A person takes socks out at random in the dark. How many socks must he take out to be sure that he has at least two blue socks?

- a) 18
- b) 35
- c) 28
- d) 14

Answer: d

Explanation: Given 12 red and 12 blue socks so, in order to take out at least 2 blue socks, first we need to take out 12 shocks (which might end up red in worst case) and then take out 2 socks (which would be definitely blue). Thus we need to take out total 14 socks.

2. The least number of computers required to connect 10 computers to 5 routers to guarantee 5 computers can directly access 5 routers is \_\_\_\_\_

- a) 74
- b) 104
- c) 30
- d) 67

Answer: c

Explanation: Since each 5 computer need directly connected with each router. So 25 connections + now remaining 5 computer, each connected to 5 different routers, so 5 connections = 30 connections.

3. In a group of 267 people how many friends are there who have an identical number of friends in that group?

- a) 266
- b) 2
- c) 138
- d) 202

Answer: b

Explanation: Suppose each of the 267 members of the group has at least 1 friend. In this case, each of the 267 members of the group will have 1 to  $267-1=266$  friends. Now, consider the numbers from 1 to  $n-1$  as holes and the  $n$  members as pigeons. Since there is  $n-1$  holes and  $n$  pigeons there must exist a hole which must contain more than one pigeon. That means there must exist a number from 1 to  $n-1$  which would contain more than 1 member. So, in a group of  $n$  members there must exist at least two persons having equal number of friends. A similar case occurs when there exist a person having no friends.

4. When four coins are tossed simultaneously, in \_\_\_\_\_ number of the outcomes at most two of the coins will turn up as heads.

- a) 17
- b) 28
- c) 11
- d) 43

Answer: c

Explanation: The question requires you to find number of the outcomes in which at most 2 coins turn up as heads i.e., 0 coins turn heads or 1 coin turns head or 2 coins turn heads. The number of outcomes in which 0 coins turn heads is  ${}^4C_0 = 1$  outcome. The number of outcomes in which 1 coin turns head is  ${}^4C_1 = 4$  outcomes. The number of outcomes in which 2 coins turn heads is,  ${}^4C_2 = 6$  outcomes. Therefore, total number of outcomes =  $1 + 4 + 6 = 11$  outcomes.

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5. How many numbers must be selected from the set  $\{1, 2, 3, 4\}$  to guarantee that at least one pair of these numbers add up to 7?

- a) 14
- b) 5
- c) 9
- d) 24

Answer: b

Explanation: With 2 elements pairs which give sum as 7 =  $\{(1,6), (2,5), (3,4), (4,3)\}$ . So choosing 1 element from each group = 4 elements (in worst case 4 elements will be either  $\{1,2,3,4\}$  or  $\{6,5,4,3\}$ ). Now using pigeonhole principle = we need to choose 1 more element so that sum will definitely be 7. So Number of elements must be  $4 + 1 = 5$ .

6. During a month with 30 days, a cricket team plays at least one game a day, but no more than 45 games. There must be a period of some number of consecutive days during which the team must play exactly \_\_\_\_\_ number of games.

- a) 17
- b) 46
- c) 124
- d) 24

Answer: d

Explanation: Let  $a_1$  be the number of games played until day 1, and so on,  $a_i$  be the no games played until  $i$ . Consider a sequence like  $a_1, a_2, \dots, a_{30}$  where  $1 \leq a_i \leq 45, \forall a_i$ . Add 14 to each element of the sequence we get a new sequence  $a_1+14, a_2+14, \dots, a_{30}+14$  where,  $15 \leq a_i+14 \leq 59, \forall a_i$ . Now we have two sequences 1.  $a_1, a_2, \dots, a_{30}$  and 2.  $a_1+14, a_2+14, \dots, a_{30}+14$ . having 60 elements in total with each elements taking a value  $\leq 59$ . So according to pigeon hole principle, there must be at least two elements taking the same value  $\leq 59$  i.e.,  $a_i = a_j + 14$  for some  $i$  and  $j$ . Therefore, there exists at least a period such as  $a_j$  to  $a_i$ , in which 14 matches are played.

7. In how many ways can 8 different dolls be packed in 5 identical gift boxes such that no box is empty if any of the boxes hold all of the toys?

- a) 2351
- b) 365
- c) 2740
- d) 1260

Answer: d

8. A group of 20 girls plucked a total of 200 oranges. How many oranges can be plucked one of them?

- a) 24
- b) 10
- c) 32
- d) 7

Answer: a

Explanation: Suppose all of them plucked the different number of oranges. A girl can pluck at least 0 oranges and the number of oranges plucked by each student is distinct. So, total number of plucked oranges should be less than 100. But  $0+1+2+\dots+19+20 = 210 > 200$  a contradiction.

Thus there exist two girls who plucked the same number of oranges. If thus there exist two girls who plucked the same number of oranges. It means each girl of remaining 18 students plucked different number of oranges. Number of oranges Plucked by 18 students =  $0+1+2+3+\dots+17 = 153$  oranges. Number of oranges plucked by remaining 2 student =  $200 - 153 = 47$ . Both students plucked same number of oranges. So, Number of oranges plucked by one of them =  $47/2=24$ .

9. In a get-together party, every person present shakes the hand of every other person. If there were 90 handshakes in all, how many persons were present at the party?

- a) 15
- b) 14
- c) 16
- d) 17

Answer: b

Explanation: Let the total number of persons present at the party be  $m$ , Then,  $\frac{m(m-1)}{2} = 90$ .  
 $m = 14$ .

10. A bag contains 25 balls such as 10 balls are red, 7 are white and 8 are blue. What is the minimum number of balls that must be picked up from the bag blindfolded (without replacing any of it) to be assured of picking at least one ball of each colour?

- a) 10
- b) 18
- c) 63
- d) 35

Answer: b

Explanation: Consider three buckets red, white and blue and we want the total number of balls such that each bucket contain at least one ball. Now consider the state of picking up a ball without replacement : (normally you consider the worst case scenario in these cases) Starting 10 balls all are red and thus goes to bucket name Red. Now again picking up the ball gives 7 balls which are of same colour and put all of them in a bucket named White. The next pick will definitely be of different colour thus: we picked  $10 + 7 + 1 = 18$ .

### **Binomial Theorem**

1. What is the general term of  $(x - y)^{xy}$ ?

- a)  $x - yCr (xxy - r . yr)$
- b)  $xyCr (xx - y - r . -yr)$
- c)  $xyCr (xxy - r . -yr)$
- d)  $x - yCr (xx - y - r . yr)$

Answer: c

Explanation: The general term of a binomial series is given by  $nCr a^n - r b^r$ .

Here  $a = x$ ,  $b = -y$  and  $n = xy$

Therefore the general term is given by  $xyCr (xxy - r . -yr)$ .



2. What is the value of  $n$ , if the coefficients of the second term of  $(x - y)^3$  is equal to the third term of the expansion  $(x + y)^n$ ?

- a)  $-2$
- b)  $3$
- c)  $4$
- d)  $5$

Answer: b

Explanation: Coefficient of the second term of  $(x - y)^3$  is  ${}^3C_1$  and the coefficient of the third term of the expansion  $(x + y)^n$  is  ${}^nC_2$ .

$${}^3C_1 = {}^nC_2$$

$$3 = \frac{n!}{2!(n-2)!}$$

$$6 = \frac{n(n-1)(n-2)!}{(n-2)!}$$

$$6 = n^2 - n$$

$$n^2 - n - 6 = 0$$

$$n^2 - 3n + 2n - 6 = 0$$

$$(n - 3)(n + 2) = 0$$

$$n = 3, -2$$

Since  $n$  cannot be negative,  $n = 3$ .

3. Which term will be the middle term of  $(xyz - x)^{2n}$ ?

- a)  $(n + 1)$ th term
- b)  $(n + 2)$ th term
- c)  $n$ th term
- d)  $(n - 1)$ th term

Answer: a

Explanation: Clearly  $2n$  is an even number and the binomial has  $2n + 1$  terms.

The middle term for a binomial with even power, is the term equal to  $(n/2 + 1)$  where  $n$  is number of terms.

In this case,  $(2n/2 + 1) = n + 1$ .

4. What is the middle term of  $(4 + 2x)^6$ ?

- a)  $11240 x^2$
- b)  $10240 x^3$
- c)  $12240 x^4$
- d)  $10340 x^4$

Answer: b

Explanation: The middle term will be the 4th term

$$4\text{th term} = {}^6C_3 (4)^6 - 3(2x)^3$$

$$= 20 (64) (8x^3)$$

$$= 10240 x^3$$

5. What is the middle term of  $(x^2 + x)^3$ ?

- a)  $3x^4$
- b)  $6x^4$
- c)  $4x^4$
- d)  $3x^6$

Answer: a

Explanation:

Since the power is odd, there will be even number of terms and two middle terms.



$r = n+12$  and  $r = n-12$  Here  $n = 3$

Therefore,  $r = 2$  and  $r = 1$ .

When  $r = 2$ ,  ${}^{3C_2} (x^2)^3 - 2(x)^2 = 3x^4$

When  $r = 1$ ,  ${}^{3C_1} (x^2)^3 - 1(x)^1 = 3x^5$

6. Which of the following values of  $n$  are possible, if the middle term of  $(x + 3y)^n$  is the fifth term.

- a) 6, 7 or 8
- b) 7, 8 or 10
- c) 7, 8 or 9
- d) 8, 9 or 10

Answer: c

Explanation: If  $n$  is the number of terms and is even, then the middle term is the  $(\frac{n}{2}+1)^{th}$  term.

Else if  $n$  is the number of terms and is odd, there are two middle terms which are the  $(\frac{n+1}{2})^{th}$  term and the  $(\frac{n+3}{2})^{th}$  term.

Case 1:  $\frac{n}{2}+1 = 5$  ;  $n = 8$

Case 2:  $\frac{n+1}{2} = 5$  ;  $n = 9$

Case 3:  $\frac{n+3}{2} = 5$  ;  $n = 7$

7. What is the even value of  $n$ , if the middle term of  $(a + b)^{2n-3}$  is 11?

- a) 12
- b) 10
- c) 20
- d) 22

Answer: a

Explanation: If  $2n-3$  is even, then the middle term is the  $(\frac{2n-3}{2}+1)^{th}$  term.

Else if  $2n-3$  is odd, there are two middle terms which are the  $(\frac{2n-3}{2}+1)^{th}$  term and the  $(\frac{2n-3}{2}+2)^{th}$  term.

Case 1:  $\frac{2n-3}{2}+1 = 11$  ;  $n = 11.5$

Case 2:  $\frac{2n-3}{2}+2 = 11$  ;  $n = 12$

Case 3:  $2n-3 = 11$  ;  $n = 11$

8. What is the value of  $n$  if the middle term  $(x + 2y)^{2n+1}$  is the 19th term?

- a) 18
- b) 19
- c) 35
- d) 37

Answer: a

Explanation: Clearly  $(2n+1)$  is an odd number. Therefore this is a case of binomial with an odd power.

For a binomial expansion with odd power, there are two middle terms.

Case 1:  $\frac{2n+1}{2}+1 = 19$

Therefore  $n = 18$

Case 2:  $\frac{2n+1}{2}+2 = 19$

Therefore  $n = 17$

9. If the general term is  ${}^{91C_2} x^{89}$ , what is the expansion?

- a)  $(x)^{91}$
- b)  $(x-2)^{90}$
- c)  $(x-1)^{91}$
- d)  $(x+1)^{90}$

Answer: c

Explanation: The general term of an expansion is  $nCr x^n - r$  yr.  
Clearly here  $n$  is 91 and the first term is  $x$  raised to the power 89.  
The second term is raised to power 2.

$$y^2 = 1$$

$$y = +1 \text{ or } -1$$

Therefore the expansion can either be  $(x + 1)^{91}$  or  $(x - 1)^{91}$ .

10. What is the middle term of  $(xyz + 3)^{80}$ ?

- a)  $80C41 (xyz)^{41} (3)^{39}$
- b)  $80C40 (xyz)^{40} (3)^{40}$
- c)  $80C39 (xyz)^{39} (3)^{40}$
- d)  $80C41 (xyz)^{41} (3)^{40}$

Answer: b

Explanation: Since the power is even, there are odd number of terms.

The middle term is the  $(\frac{n}{2}+1)^{th}$  term.

$$= (80/2+1)^{th} \text{ term}$$

$$= 41^{st} \text{ term}$$

$$\text{The } 41^{st} \text{ term} = 80C40 (xyz)^{40} (3)^{40}$$

11. What is the coefficient of the middle term of  $(z + y)^{3x}$ , if  $3x$  is considered to be even and the middle term is the 4th term?

- a)  $7C3$
- b)  $6C2$
- c)  $6C3$
- d)  $7C2$

Answer: c

Explanation: Since the middle term is the fourth term

$$\text{Considering } 3x \text{ to be even, } (\frac{3x}{2}+1)^{th} \text{ term} = 4$$

$$x = 6/3$$

Therefore, the fourth term coefficient is  $6C3$

12. What is the fourth term of  $(x - 5y)^{96}$ ?

- a)  $125 \cdot 96C3 x^{93} y^3$
- b)  $625 \cdot 96C3 x^{93} y^4$
- c)  $625 \cdot 96C4 x^{92} y^4$
- d)  $125 \cdot 96C4 x^{92} y^4$

Answer: a

$$\text{Explanation: } T_{r+1} = nCr x^n - r y^r$$

Here first term is 4 and second term is  $5y$ .

$$n = 96$$

$$r = 3$$

$$\text{Therefore, } T_{r+1} = 96C3 x^{96-3} - 3 (5y)^3$$

$$= 125 \cdot 96C3 x^{93} y^3$$

### Mathematical induction

1. What is the base case for the inequality  $7^n > n^3$ , where  $n = 3$ ?

- a)  $652 > 189$
- b)  $42 < 132$

- c)  $343 > 27$   
 d)  $42 \leq 431$

Answer: c

Explanation: By the principle of mathematical induction, we have  $73 > 33 \Rightarrow 343 > 27$  as a base case and it is true for  $n = 3$ .

2. In the principle of mathematical induction, which of the following steps is mandatory?

- a) induction hypothesis  
 b) inductive reference  
 c) induction set assumption  
 d) minimal set representation

3. For  $m = 1, 2, \dots$ ,  $4m+2$  is a multiple of \_\_\_\_\_

- a) 3  
 b) 5  
 c) 6  
 d) 2

Answer: d

Explanation: For  $n = 1$ ,  $4 * 1 + 2 = 6$ , which is a multiple of 2. Assume that  $4m+2$  is true for  $m=k$  and so  $4k+2$  is true based on the assumption. Now, to prove that  $4k+2$  is also a multiple of 2  $\Rightarrow 4(k+1)+2 \Rightarrow 2 * 4k - 4k + 6 \Rightarrow 2*4k+4 - 4k+2 \Rightarrow 2(4k+2) - 2(2k+1)$ . Here, the first term  $2(4k+2)$  is true as per assumption and the second term  $2(2k+1)$  is must to be a multiple of 2. Hence,  $4(k+1)+2$  is a multiple of 2. So, by induction hypothesis,  $(4m+2)$  is a multiple of 2, for  $m = 1, 2, 3, \dots$

4. For any integer  $m \geq 3$ , the series  $2+4+6+\dots+(4m)$  can be equivalent to \_\_\_\_\_

- a)  $m^2+3$   
 b)  $m+1$   
 c)  $mm$   
 d)  $3m^2+4$

Answer: a

Explanation: The required answer is  $m^2+3$ . Now, by induction assumption, we have to prove  $2+4+6+\dots+4(k+1) = (k+1)^2+3$  also can be true,  $2+4+6+\dots+4(k+1) = 2+4+6+\dots+(4k+4)$  and by the subsequent steps, we can prove that  $(m+1)^2+3$  also holds for  $m=k$ . So, it is proved.

5. For every natural number  $k$ , which of the following is true?

- a)  $(mn)^k = m^k n^k$   
 b)  $m^k = n + 1$   
 c)  $(m+n)^k = k + 1$   
 d)  $mkn = mnk$

Answer: a

Explanation: In the first step, for  $k = 1$ ,  $(mn)^1 = m^1 n^1 = mn$ , hence it is true. Let us assume the statement is true for  $k = l$ , Now by induction assumption,  $(mn)^l = m^l n^l$  is true. So, to prove,  $(mn)^{l+1} = m^{l+1} n^{l+1}$ , we have  $(mn)^l = m^l n^l$  and multiplying both sides by  $(mn) \Rightarrow (mn)^l (mn) = (m^l n^l) (mn) \Rightarrow (mn)^{l+1} = (m^l m) (n^l n) \Rightarrow (mn)^{l+1} = (m^{l+1} n^{l+1})$ . Hence, it is proved. So,  $(mn)^k = m^k n^k$  is true for every natural number  $k$ .

6. By induction hypothesis, the series  $12 + 22 + 32 + \dots + p^2$  can be proved equivalent to

- a)  $p^2 + 27$
- b)  $p \cdot (p+1) \cdot (2p+1) / 6$
- c)  $p \cdot (p+1) / 4$
- d)  $p + p^2$

Answer: b

Explanation: By principle of mathematical induction, we now assume that p (b) is true  $12 + 22 + 32 + \dots + b^2 = b(b+1)(2b+1) / 6$

so to prove P(b+1):  $12 + 22 + 32 + \dots + b^2 + (b+1)^2 = b(b+1)(2b+1) / 6 + (b+1)^2$

By induction assumption it is shown that  $12 + 22 + 32 + \dots + b^2 + (b+1)^2 = (b+1)[(b+2)(2b+3)] / 6$ .

Hence it is proved that  $12 + 22 + 32 + \dots + p^2 = p \cdot (p+1) \cdot (2p+1) / 6$ .

7. For any positive integer m \_\_\_\_\_ is divisible by 4.

- a)  $5m^2 + 2$
- b)  $3m + 1$
- c)  $m^2 + 3$
- d)  $m^3 + 3m$

Answer: d

Explanation: The required answer is,  $m^3 + 3m$ . Now, by induction hypothesis, we have to prove for  $m=k$ ,  $k^3 + 3k$  is divisible by 4. So,  $(k+1)^3 + 3(k+1) = k^3 + 3k^2 + 6k + 4$   
 $= [k^3 + 3k] + [3k^2 + 3k + 4] = 4M + (12k^2 + 12k) - (8k^2 + 8k - 4)$ , both the terms are divisible by 4.  
Hence  $(k+1)^3 + 3(k+1)$  is also divisible by 4 and hence it is proved for any integer m.

8. According to principle of mathematical induction, if  $P(k+1) = m(k+1) + 5$  is true then \_\_\_\_\_ must be true.

- a)  $P(k) = 3m(k)$
- b)  $P(k) = m(k) + 5$
- c)  $P(k) = m(k+2) + 5$
- d)  $P(k) = m(k)$

Answer: b

Explanation: By the principle of mathematical induction, if a statement is true for any number  $m = k$ , then for its successor  $m = k + 1$ , the statement also satisfies, provided the statement is true for  $m = 1$ . So, the required answer is  $p(k) = mk + 5$ .

9. Which of the following is the base case for  $4n+1 > (n+1)^2$  where  $n = 2$ ?

- a)  $64 > 9$
- b)  $16 > 2$
- c)  $27 < 91$
- d)  $54 > 8$

Answer: a

Explanation: Statement By principle of mathematical induction, for  $n=2$  the base case of the inequation  $4n+1 > (n+1)^2$  should be  $64 > 9$  and it is true.

10. What is the induction hypothesis assumption for the inequality  $m! > 2m$  where  $m \geq 4$ ?

- a) for  $m=k$ ,  $k+1! > 2k$  holds
- b) for  $m=k$ ,  $k! > 2k$  holds
- c) for  $m=k$ ,  $k! > 3k$  holds
- d) for  $m=k$ ,  $k! > 2k+1$  holds



Answer: b

Explanation: By the induction hypothesis, assume that  $p(k) = k! > 2k$  is true, for  $m=k$  and we need to prove this by the principle of mathematical induction.

### **Boolean Algebra**

1. Algebra of logic is termed as \_\_\_\_\_

- a) Numerical logic
- b) Boolean algebra
- c) Arithmetic logic
- d) Boolean number

Answer: b

Explanation: The variables that can have two discrete values False(0) and True(1) and the operations of logical significance are dealt with Boolean algebra.

2. Boolean algebra can be used \_\_\_\_\_

- a) For designing of the digital computers
- b) In building logic symbols
- c) Circuit theory
- d) Building algebraic functions

Answer: a

Explanation: For designing digital computers and building different electronic circuits boolean algebra is accepted widely.

3. What is the definition of Boolean functions?

- a) An arithmetic function with  $k$  degrees such that  $f:Y \rightarrow Y^k$
- b) A special mathematical function with  $n$  degrees such that  $f:Y^n \rightarrow Y$
- c) An algebraic function with  $n$  degrees such that  $f:X^n \rightarrow X$
- d) A polynomial function with  $k$  degrees such that  $f:X^2 \rightarrow X^n$

Answer: b

Explanation: A Boolean function is a special mathematical function with  $n$  degrees and where  $Y = \{0,1\}$  is the Boolean domain with  $n$  being a non-negative integer. It helps in describing the way in which the Boolean output is derived from Boolean inputs.

4.  $F(X,Y,Z,M) = X'Y'Z'M'$ . The degree of the function is \_\_\_\_\_

- a) 2
- b) 5
- c) 4
- d) 1

Answer: c

Explanation: This is a function of degree 4 from the set of ordered pairs of Boolean variables to the set  $\{0,1\}$ .

5. A \_\_\_\_\_ value is represented by a Boolean expression.

- a) Positive
- b) Recursive
- c) Negative

d) Boolean

Answer: d

Explanation: A Boolean value is given by a Boolean expression which is formed by combining Boolean variables and logical connectives.

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6. Which of the following is a Simplification law?

- a)  $M.(\sim M+N) = M.N$
- b)  $M+(N.O) = (M+N)(M+O)$
- c)  $\sim(M+N) = \sim M.\sim N$
- d)  $M.(N.O) = (M.N).O$

Answer: a

Explanation: By Simplification Law we can have  $X.(\sim X+Y) = X.Y$  and  $X+(\sim X.Y) = X+Y$ . By De' Morgan's law  $\sim(X+Y) = \sim X.\sim Y$ . By commutative law we can say that  $A.(B.C) = (A.B).C$ .

7. What are the canonical forms of Boolean Expressions?

- a) OR and XOR
- b) NOR and XNOR
- c) MAX and MIN
- d) SOM and POM

Answer: d

Explanation: There are two kinds of canonical forms for a Boolean expression-> 1)sum of minterms(SOM) form and

2)product of maxterms(SOM) form.

8. Which of the following is/are the universal logic gates?

- a) OR and NOR
- b) AND
- c) NAND and NOR
- d) NOT

Answer: c

Explanation: NAND and NOR gates are known as the universal logic gates. A universal gate is a gate which can implement any Boolean function without the help of 3 basic gate types.

9. The logic gate that provides high output for same inputs \_\_\_\_\_

- a) NOT
- b) X-NOR
- c) AND
- d) XOR

Answer: b

Explanation: The logic gate which gives high output for the same inputs, otherwise low output is known as X-NOR or Exclusive NOR gate.

10. The \_\_\_\_\_ of all the variables in direct or complemented form is a maxterm.

- a) addition
- b) product
- c) moduler
- d) subtraction



Answer: a

Explanation: The Boolean function is expressed as a sum of the 1-minterms and the inverse of function is represented as 0-minterms.

1. What is the correct order of operations to be performed, assuming that there are no parenthesis?

- a) Complementation, followed by AND and then OR
- b) OR, followed by Complementation and then AND
- c) AND, followed by Complementation and then OR
- d) Complementation, followed by OR and then AND

Answer: a

2. How many rows will a truth table for an n-variable expression have?

- a) n
- b)  $n^2$
- c)  $2^n$
- d)  $2n$

Answer: c

Explanation: Because the expression has n variables, and each variable can have the value 0 or 1, the number of different combinations of values of the variables is  $2 \times 2 \times \dots = 2^n$  times.

### **Linear Algebra**

3. Which one of the following is not a criterion for linearity of an equation?

- a) The dependent variable y should be of second order
- b) The derivatives of the dependent variable should be of second order
- c) Each coefficient does not depend on the independent variable
- d) Each coefficient depends only on the independent variable

Answer: c

Explanation: The two criteria for linearity of an equation are: The dependent variable y and its derivatives of first degree. Each coefficient depends only on the independent variable.

5. Which of the following is true for matrices?

- a)  $(AB)^{-1} = B^{-1}A^{-1}$
- b)  $(AT)^T = A$
- c)  $AB = BA$
- d)  $A \cdot I = I$

Answer: a

4. Which among the following does not belong to main types of integrals?

- a) Indefinite Integral
- b) Proper Definite Integral
- c) Improper Definite Integral
- d) Real Integral

Answer: d

### **Probability**



4. Given that E and F are events such that  $P(E) = 0.5$ ,  $P(F) = 0.4$  and  $P(E \cap F) = 0.3$ , then what will be the value of  $P(F|E)$ ?

- a)  $2/5$
- b)  $3/5$
- c)  $3/4$
- d)  $2/4$

Answer: b

Explanation: We know that  $P(F|E) = P(E \cap F) / P(E)$ . (By formula for conditional probability)

Value of  $P(E \cap F)$  is given to be 0.3 and value of  $P(E)$  is given to be 0.5.

$$P(F|E) = (0.3) / (0.5).$$

$$P(F|E) = 3 / 5.$$

2. Mean of a random variable X is given by \_\_\_\_\_

- a)  $E(X)$
- b)  $E(X^2)$
- c)  $E(X^2) - (E(X))^2$
- d)  $(E(X))^2$

Answer: a

3. Variance of a random variable X is given by \_\_\_\_\_

- a)  $E(X)$
- b)  $E(X^2)$
- c)  $E(X^2) - (E(X))^2$
- d)  $(E(X))^2$

Answer: c

4. Mean of a constant 'a' is \_\_\_\_\_

- a) 0
- b) a
- c)  $a/2$
- d) 1

Answer: b

5. Variance of a constant 'a' is \_\_\_\_\_

- a) 0
- b) a
- c)  $a/2$
- d) 1

Answer: a

11. What is moment generating function?

- a)  $M_x(t) = E(e^{tx})$
- b)  $M_x(t) = E(e^{-tx})$
- c)  $M_x(t) = E(e^{2tx})$
- d)  $M_x(t) = E(e^t)$

Answer: a



1. Method in which the previously calculated probabilities are revised with values of new probability is called \_\_\_\_\_

- a) Revision theorem
- b) Bayes theorem
- c) Dependent theorem
- d) Updation theorem

Answer: b

4. Previous probabilities in Bayes Theorem that are changed with the new available information are called \_\_\_\_\_

- a) independent probabilities
- b) dependent probabilities
- c) interior probabilities
- d) posterior probabilities

Answer: d

5. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

- a)  $1/8$
- b)  $5/8$
- c)  $2/7$
- d)  $3/8$

Answer: d

6. Bag 1 contains 3 red and 5 black balls while another Bag 2 contains 4 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it is drawn from bag 2.

- a)  $31/62$
- b)  $16/62$
- c)  $16/31$
- d)  $31/32$

Answer: c

7. Bag 1 contains 4 white and 6 black balls while another Bag 2 contains 4 white and 3 black balls. One ball is drawn at random from one of the bags and it is found to be black. Find the probability that it was drawn from Bag 1.

- a)  $12/13$
- b)  $5/12$
- c)  $7/11$
- d)  $7/12$

Answer: d

2. Find which of the following is a Continuous random variable?

- a) Number of kids in a family
- b) Number of planets around the sun
- c) Number of tails tossing a coin four times
- d) Life of an electric fan

Answer: d

3. Find the value of  $P(X=3)$  if  $X$  is the discrete random variable taking values  $x_1, x_2, x_3$  where  $P(X=0)=0$ ,  $P(X=1) = 1/4$  and  $P(X=2) = 1/4$

- a) 1
- b)  $1/2$
- c)  $1/3$
- d)  $1/4$

Answer: b

6. Determine the value  $c$  so that the following function can serve as a probability distribution of the discrete random variable  $x$ :

$f(x)=c(x+4)$ , for  $x=0,1,2,3$

- a)  $1/20$
- b)  $1/16$
- c)  $1/18$
- d)  $1/22$

Answer: d

7. A dice is thrown, what is the probability of getting an odd number?

- a)  $1/8$
- b)  $1/6$
- c)  $1/2$
- d)  $1/4$

Answer: c

4. Mutually Exclusive events \_\_\_\_\_

- a) Contain all sample points
- b) Contain all common sample points
- c) Does not contain any sample point
- d) Does not contain any common sample point

Answer: d

7. A variable that can assume any value between two given points is called \_\_\_\_\_

- a) Continuous random variable
- b) Discrete random variable
- c) Irregular random variable
- d) Uncertain random variable

Answer: a

9.  $X$  is a variate between 0 and 3. The value of  $E(X^2)$  is \_\_\_\_\_

- a) 8
- b) 7
- c) 27
- d) 9

Answer: d



1. If 40% of boys opted for maths and 60% of girls opted for maths, then what is the probability that maths is chosen if half of the class's population is girls?

- a) 0.5
- b) 0.6
- c) 0.7
- d) 0.4

Answer: a

2. Company A produces 10% defective products, Company B produces 20% defective products and C produces 5% defective products. If choosing a company is an equally likely event, then find the probability that the product chosen is defective.

- a) 0.22
- b) 0.12
- c) 0.11
- d) 0.21

Answer: b

6. Total probability theorem is used in Baye's theorem.

- a) True
- b) False

Answer: a

5. The probability that the political party A does a particular work is 30% and the political party B doing the same work is 40%. Then find the probability that the work is completed if the probability of choosing the political party A is 40% and that of B is 60%.

- a) 0.12
- b) 0.24
- c) 0.36
- d) 0.48

Answer: c

Explanation: Let A be the event of completing the work and  $E_i$  be the event of selecting the political party. Then,

$$\begin{aligned} P(A) &= P(E_1) P(A|E_1) + P(E_2) P(A|E_2) \\ &= (40/100)(30/100) + (60/100)(40/100) \\ &= 0.36. \end{aligned}$$

10. Let there be two newly launched phones A and B. The probability that phone A has good battery life is 0.7 and the probability that phone B has good battery life is 0.8. Then find the probability that a phone has a good battery life.

- a) 0.65
- b) 0.75
- c) 0.85
- d) 0.45

Answer: b

5. A coin toss is an example of \_\_\_\_\_ random variable.

- a) continuous

- b) discrete
- c) either continuous or discrete
- d) mixed

Answer: b

6. The weather can be sunny, cloudy or rainy. All three are equally likely. What is the probability that the weather is sunny?

- a) 1
- b) 0
- c)  $1/3$
- d)  $2/3$

Answer: c

10. The full form of CDF in relation to probability theory is \_\_\_\_\_

- a) Continuous Distribution Function
- b) Cumulative Distribution Function
- c) Cumulative Distributive Function
- d) Continuous Distributive Function

Answer: b

7. How many types of discrete random variables are there?

- a) Two
- b) Three
- c) Four
- d) Five

Answer: a

9.  $Y = X$  for  $0 < X < 1$ .  $Y$  is \_\_\_\_\_ random variable.

- a) mixed
- b) discrete
- c) continuous
- d) non-numerical

Answer: c

10. The mean of binomial distribution is \_\_\_\_\_.

- (a)  $nq$
- (b)  $pq$
- (c)  $np$
- (d) one of the above

Answer: Option (c)

1. In a Poisson Distribution, if 'n' is the number of trials and 'p' is the probability of success, then the mean value is given by?

- a)  $m = np$

- b)  $m = (np)^2$
- c)  $m = np(1-p)$
- d)  $m = p$

Answer: a

5. In a Poisson Distribution, the mean and variance are equal.

- a) True
- b) False

Answer: a

3. What is the value of chebyshev polynomial of degree 0?

- a) 1
- b) 0
- c) -1
- d) 2

Answer: a

8. Chebyshev polynomials of odd orders are \_\_\_\_\_

- a) Even functions
- b) Odd functions
- c) Exponential functions
- d) Logarithmic functions

Answer: b

28. If E and F are two events such that  $P(a) = 0.2$ ,  $P(b) = 0.6$  and  $P(E/F) = 0.2$  then the value of  $P(E/\sim F)$  is \_\_\_\_\_

- a) 0.8
- b) 0.2
- c)  $1/3$
- d) 0.5

Answer: b

Explanation: For independent events,  $P(E/\sim F) = P(a) = 0.2$ .

26. A programmer has a 95% chance of finding a bug every time she compiles his code, and it takes her three hours to rewrite the code every time she discovers a bug. Find the probability that she will finish her program by the end of her workday. (Assume that a workday is 9 hours)

- a) 44%
- b) 76%
- c) 28%
- d) 37%

Answer: c

